Games, graphs, and machines

Partial orders 3

n	a(n)
0	1
1	1
2	3
3	19
4	219
5	4231
6	130023
7	6129859
8	431723379
9	44511042511
10	6611065248783
11	1396281677105899
12	414864951055853499
13	171850728381587059351
14	98484324257128207032183
15	77567171020440688353049939
16	83480529785490157813844256579
17	12215254125029532286294128126915
18	2419393925972011766028978201480850

$a(n)$ = Number of partial orders on $\{1, 2, \ldots, n\}$.

$a(2)$

Chains

distinct

A *chain* in a poset is a sequence of elements $a_1, \ldots, a_n$ such that

$$a_1 \preceq a_2 \preceq \cdots \preceq a_n.$$

The number n is the *length* of the chain.

Find a chain of length 3 in the subset poset of $\{1, 2, 3, 4\}$.

Set of all subsets of $\{1,2,3,4\}$

$a_1 \leq a_2 < a_3$

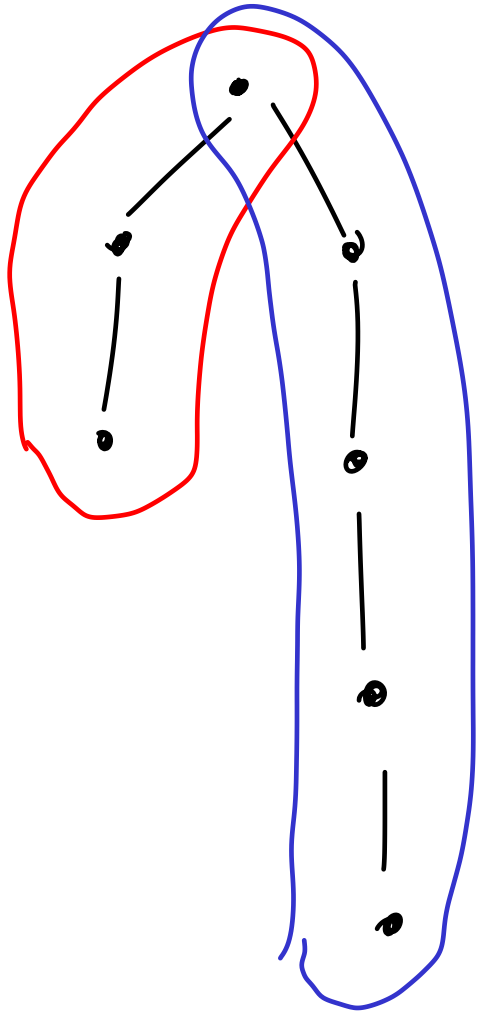
$\phi \rightarrow \{1\} \rightarrow \{1,2\} \rightarrow \{1,2,3\} \rightarrow \{1,2,3,4\}$

$\phi \rightarrow \{2\} \nearrow$

Maximal chains

- What could be the meaning of a *maximal chain*?
- Find a maximal chain in the subset poset of $\{1, 2, 3, 4\}$.

maximal chain = chain that cannot be extended.



Both

maximal

A theorem

A poset in which all maximal chains have the same (finite) length is called a *graded poset*.

Theorem
If a poset is graded poset, then it has a rank function.

Verify the theorem for the subset poset of $\{1, \ldots, n\}$.

} finite

$n=4$

If A then B

A ? — NO ⇒ Nothing to check

YES ⇒ Check B.

In a max chain a_i must have $i-1$ elts.

$a_1 \lessdot a_2 \lessdot \ldots$

B: Take $a \mapsto |a|$.

Converse?

Theorem
If a poset is graded poset, then it has a rank function.

The *converse* of "If A then B" is "If B then A".

Is the converse of the theorem true?